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A RE-DETERMINATION OF THE VALUE OF THE ELECTRON AND OF RELATED CONSTANTS

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This re-determination of the most fundamental of physical constants was entered upon three years ago for three reasons.

First, in 1913 results began to be published from Vienna,[1] which though obtained by a modification of my method,[2] were wholly irreconcilable with those which I had found; and I accordingly wished to see whether I could find conditions under which the method failed.

Second, there developed a tendency, especially among British physicists, to adopt a value of e about 2% lower than that which I had obtained, and as this difference was much greater than the necessary error in my method I was anxious to see, by entirely new work, whether a numerical error could have crept into the former determination.

Third, the electron has recently taken on added importance because it has been found to carry with it not merely all molecular and atomic magnitudes, as heretofore, but also all of the most significant of the radiation constants, such as Planck's h, the Stefan-Boltzmann constant σ, the Wien constant C_2, all X-ray constants, i.e., the wave lengths of characteristic X-rays, etc.[3] It seemed worth while therefore to drive my method, which is certainly exceedingly exact if its validity is granted, to the utmost limit of its possible precision.

The method is the same as that used in the preceding determination,[4] but the apparatus is new throughout and every constant entering into the value of e has been redetermined with increased care and precision. The condenser plates MN (fig. 1) consist of two optically flat brass surfaces 22 cm. in diameter, held apart by three small pieces of echelon plates about 1 cm. square and 14.9174 mm. thick placed at points 60°

apart about the circumference. The dimensions of the condenser therefore now introduce an error of no more than 1 part in 10,000. The oil droplets from the atomizer A blown by a puff of air through r entered the condenser MN through 5 minute holes 0.25 mm. in diameter in the middle of the upper plate and were observed by means of light from the arc a, filtered through a trough of water w and one of cupric chloride d for the removal of heat rays. The temperature was held constant to within one or two hundredths of a degree by the oil-bath G. The charge on the drop p was changed by X-rays from the bulb X passing through the window g. The pressure was varied from 13 cm. to 76

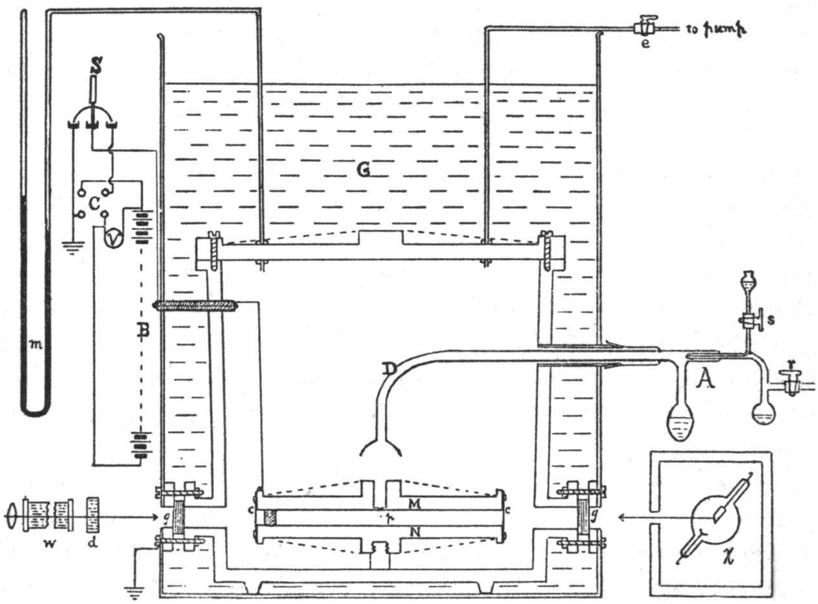

FIG. 1.

cm. and was measured to 0.1 mm. by the manometer m. The atomizer A, fed from s with the highest grade of watch-oil, density at 23°C. redetermined as 0.9199, was blown with carefully dried and cleaned air let in through the cock r, the bulbs below A being to catch excess oil. The observing optical system was a specially constructed telescope of 30 mm. objective and a magnification of 25 diameters. The distance through which the drops were timed (cross hair distance) was 1.0220 cm. correct to 1 part in 2000. The velocities of the drops were measured with a most convenient and reliable printing chronograph made by Wm. Gaertner & Company of 5545 Lake Avenue, Chicago, and kindly loaned to the laboratory for this determination. It prints directly the time of pushing a key, in terms of the indications of a standard clock, with an error which is never more than 1/100 second. The times of fall and rise were from 14 to 60 seconds. This makes the error in the mean of a series of time determinations a wholly negligible quantity. The electric field strengths were measured to 1 part in 3000, by a 750 volt Weston standard laboratory voltmeter calibrated at frequent intervals during the experiment against 3 standard Weston cells. The coefficient of viscosity of air was redetermined with extraordinary precision by Dr. E. L. Harrington who, using the constant deflection apparatus designed by Dr. Gilchrist and myself, succeeded in introducing such improvements in conditions and perfections in detail as to make his final value altogether unique in its reliability and precision.[5] In view of this and other work now in progress with the same apparatus it is hardly possible that the correct value of η for dry air at 23°C. can be more than 1 part in 2000 removed from the value 0.00018227. This is within less than 0.1% of my former value, viz., 0.0001824.[6] All the other elements of the problem have been looked to with a care which is the outgrowth of six years of experience with measurements of this kind.

That portion of the investigation which has had to do with the testing of the general validity of the method has been reported in detail elsewhere.[7] Suffice it to say here that I find no indications whatever that, when properly used, it ever fails, or that it ever even remotely suggests the existence of a subelectron.

The precision of the method is sufficiently attested by the consistency of the results on different drops, provided no constant error inheres in the measurement of the dimensions of the condenser, the volts, the time, or the viscosity of air. The extent of this consistency is shown in the figure and the table which present the observations on 25 consecutive drops taken with all possible precautions during a period of several months. The data on these drops are treated precisely in the manner adopted in the 1913 article. A more detailed presentation of this work will be published elsewhere. It will be seen from the table that the final mean value of $e^{\frac{2}{3}}$ is 61.126×10^{-8}. *There is but one drop in the table which yields a value of* $e^{\frac{2}{3}}$ *differing from this by as much as one-third of one per cent and the probable error of the mean computed by least squares is one part in 4000.* This value is 0.07% higher than the value 61.086, which I published in 1913. Both values however are computed in terms of 0.0001824 as the coefficient of viscosity of air. The new value 0.00018227 is more reliable than the old and is 0.07% lower *so that the new value of e computed solely from the new data obtained in this re-*

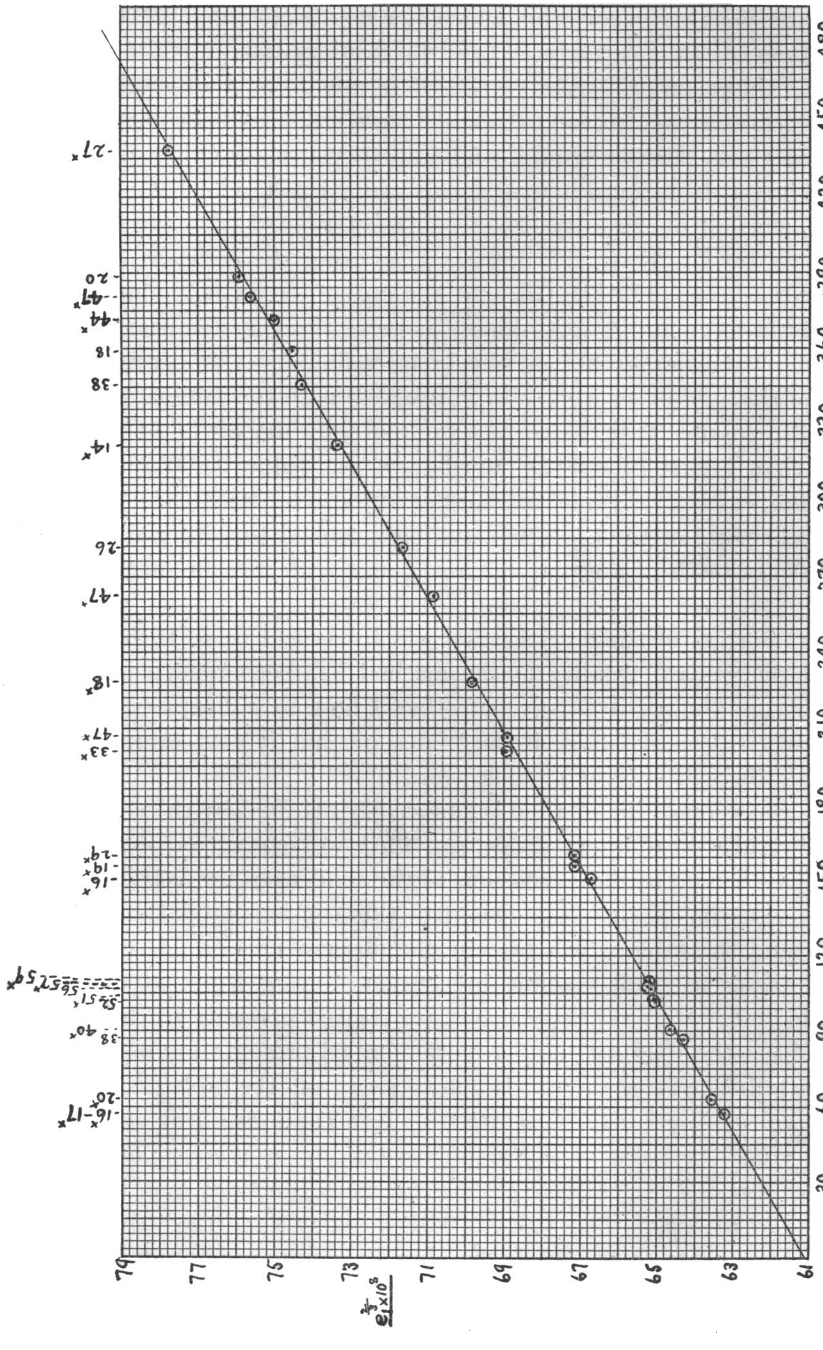
$\frac{1}{pa}$
480
450
420
390
360
330
300
270
240
210
180
150
120
90
60
30
79
77
75
73
71
69
67
65
63
61

FIG. 2.

determination is exactly the same as the value published in 1913. The uncertainty in this value should now be no more than 1 part in 1000 for it now contains but two factors which are uncertain by as much as 1 part in 2000, namely the coefficient of viscosity of air and the distance of fall of the drops (cross hair distance). The result of this final work on e may then be stated thus:

$$e = 4.774 \times 10^{-10} \pm 0.005.$$

The values of the most important radiation constants may be found from the value of e as follows:

TABLE

NO.	$\frac{1}{pa}$	$e_1^{2/3} \times 10^8$	$e^{2/3} \times 10^8$	NO.	$\frac{1}{pa}$	$e_1^{2/3} \times 10^8$	$e^{2/3} \times 10^8$
1	57.45	63.21	61.03	14	206.4	68.90	61.11
2	57.5	63.204	61.03	15	200.7	68.97	61.39
3	63.0	63.54	61.16	16	227.8	69.88	61.27
4	86.7	64.27	60.97	17	262.4	70.85	60.94
5	90.6	64.63	61.21	18	281.4	71.60	60.98
6	101.3	65.02	61.19	19	321.4	73.34	61.20
7	102.4	65.07	61.20	20	345.4	74.27	61.22
8	106.3	65.13	61.11	21	359.1	74.54	60.97
9	109.7	65.19	61.05	22	371.5	75.00	60.97
10	107.3	65.21	61.16	23	380.6	75.62	61.24
11	150.6	66.70	61.01	24	388.5	75.92	61.24
12	160.1	67.12	61.07	25	438.3	77.74	61.18
13	155.6	67.14	61.26				

Mean—61.126.

Bohr's theory[8] gives the constant of the Balmer series in hydrogen as

$$\frac{2\pi^2 e^4 m}{h^3} \text{ or } \frac{2\pi^2 e^5}{h^3 \frac{e}{m}}. \tag{1}$$

I have directly determined h photoelectrically[9] with an error of no more than 0.5%, the value from my sodium curve coming out 6.56×10^{-27}. Webster's[10] value found by the method discovered by Duane and Hunt[11] is 6.53×10^{-27}. From the mean of these two recent determinations of h, the foregoing value of e, and $e/m = 1.767 \times 10^7$, the Rydberg constant is found by (1) to be 3.294×10^{15}. Its experimental value known with the great precision attained in all wave length measurements is 3.290×10^{15}. This agreement constitutes most extraordinary justification of Bohr's equation and warrants the use of this spectroscopic data, combined with the foregoing data on e for a most exact evaluation of h. The value thus obtained is

$$h = 6.547 \times 10^{27} \pm 0.011.$$

This value should involve an uncertainty of but 1 part in 600. It will be seen to agree within 1 part in 500 with the value obtained directly from my sodium curve—a value which I estimated correct to only 1 part in 200.

Again the Wien constant C_2 of spectral radiation is given by [12]

$$C_2 = \frac{hc}{k} = \frac{6.547 \times 10^{-27} \times 2.999 \times 10^{10}}{1.372 \times 10^{-16}} = 1.4312 \pm .0030 \text{ cm. degrees.}$$

The estimated uncertainty is obtained from an uncertainty of 1 part in 600 for h and 1 part in 1000 or k. The latest Reichsanstalt value[13] is $C_2 = 1.4300$, while Coblentz[14] obtains from two different modes of approach $C_2 = 1.4322$ and 1.4369. Again from Planck's equations $C_2 = (48\,\pi\sigma k/a)^{\frac{1}{3}}$ and $\sigma = ac/4$ and the foregoing values of k and C_2 computed from e we obtain

$$\sigma = 5.72 \times 10^{-12} \pm 0.034 \text{ watt cm.}^{-2} \text{ deg.}^{-4},$$

which is exactly the result recently found by Coblentz.[14]

A summary of the most important constants the values of which are fixed by the foregoing determination of e is given below with the uncertainty attaching to each.

The Electron	e	$= 4.774 \pm 0.005 \times 10^{-10}$
The Avogadro Constant	N	$= 6.062 \pm 0.006 \times 10^{23}$
Number of gas molecules per cc. at 0°C 76 cm.	n	$= 2.705 \pm 0.003 \times 10^{19}$
Kinetic energy of translation of a molecule at 0°C.	E_0	$= 5.621 \pm 0.006 \times 10^{-14}$
Change of translational molecular energy per °C.	ϵ	$= 2.058 \pm 0.002 \times 10^{-16}$
Mass of an atom of hydrogen	m	$= 1.662 \pm 0.002 \times 10^{-24}$
Planck's element of action	h	$= 6.547 \pm 0.013 \times 10^{-27}$
Wien constant of spectral radiation	C_2	$= 1.4312 \pm 0.0030$
Stefan-Boltzmann constant of total radiation	σ	$= 5.72 \pm 0.034 \times 10^{-12}$
Grating spacing in calcite[15]	d	$= 3.030 \pm 0.001$ A

[1] Ehrenhaft, F., *Ann. Physik, Leipzig*, **44**, 1914, (657); **46**, 1915, (261); also *Physik. Zs. Leipzig*, **16**, 1915, (10).

[2] Millikan, R. A., *Physic. Rev., Ithaca*, **32**, 1911, (349–397); (Ser. 2), **2**, 1913, (109–143).

[3] Millikan, R. A., *Ibid.*, (Ser. 2), **7**, 1916, (353–388); and Webster, D. B., (607).

[4] Millikan, R. A., *Ibid.*, (Ser. 2), **2**, 1913, (109–143).

[5] Harrington, E. L., *Ibid.*, (Ser. 2), December, 1916.

[6] Millikan, R. A., *Ann. Physik., Leipzig*, **41**, 1913, (759).

[7] Millikan, R. A., *Physic. Rev., Ithaca*, December, 1916.

[8] Millikan, R. A., *Ibid.*, (Ser. 2), **7**, 1916, (374).

[9] Bohr, N., *Phil. Mag., London*, **26**, 1913, (1).

[10] Webster, *Physic. Rev., Ithaca*, (Ser. 2), 1916, (599).

[11] Duane, and Hunt, *Ibid.*, **6**, 1915, (166).

[12] Millikan, R. A., *Ibid.*, **2**, 1913, (142).

[13] Mueller, Warburg, et al., *Ann. Physik., Leipzig*, **48**, 1915, (430).

[14] Coblentz, *Physic. Rev., Ithaca*, (Ser. 2), **7**, 1916, (694).

[15] Webster, D. B., *Ibid.*, **7**, 1916, (607).